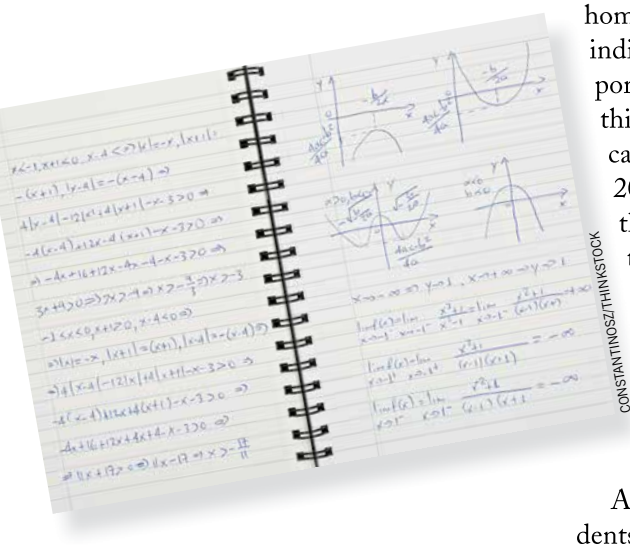
Making the Most of Going over

Samuel Otten, Michelle Cirillo, and Beth A. Herbel-Eisenmann

According to two studies of middle school and high school mathematics classrooms, 15 to 20 percent of class time tends to be spent reviewing homework (Grouws et al. 2010; Otten, Herbel-Eisenmann, and Cirillo 2012). So how can class time spent going over homework (GOHW) provide students with rich opportunities to learn from their homework? What are some ways that go beyond the opportunities embedded in the assignment itself?

An important characteristic of homework is that it provides each individual student with the opportunity to develop skills and to think about important mathematical ideas (Wieman and Arbaugh 2014). In class, GOHW provides the complementary opportunity to discuss those ideas collectively. This article focuses on the discourse—the use of spoken or written language as well as other modes of communication to convey meaning—of GOHW.



CONSTANTINOSZ/THINKSTOCK

As pointed out by NCTM, students' active engagement in mathematical discourse is especially important because collectively developing meaning supports learning (NCTM 2014; Cirillo et al. 2014; Smith et al. 2009) and distributes mathematical authority to the students (Webel 2010). Moreover, rich discourse goes hand in hand with the Standards for Mathematical Practice (SMP) from the Common Core

PURESTOCK/THINKSTOCK

Homework

Reconsider typical discourse strategies when discussing homework and move toward a system that promotes the Standards for Mathematical Practice.

State Standards for Mathematics (CCSSI 2010) because mathematical discussions allow students to share and critique reasoning, look for and express structure or patterns, and develop abstract concepts (Koestler et al. 2013).

In the sections below, we describe discourse patterns that seem to be typical of GOHW. We point out some limitations that are inherent in these typical discourse patterns and argue that GOHW can be time well spent, making unique and powerful contributions to students' learning opportunities. However, this will occur only if we shift the discourse patterns in purposeful ways. Specifically, we propose strategies for going over homework that create opportunities for students

to engage in the Common Core's Mathematical Practices.

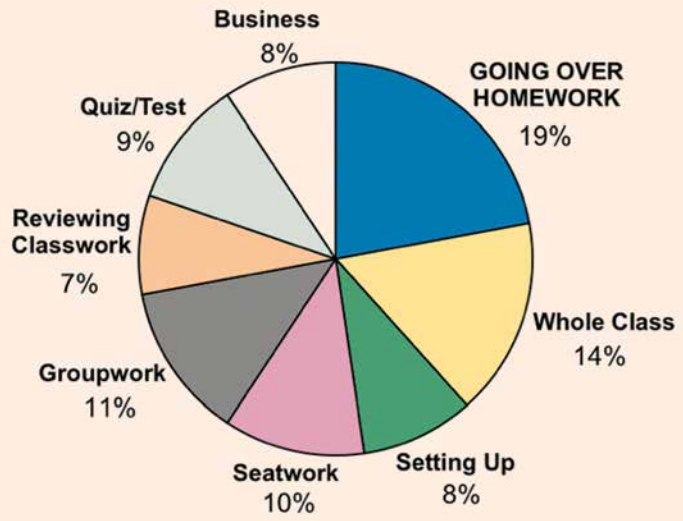
COMMON DISCOURSE PATTERNS

Our findings about the patterns of GOHW are based on our analysis of 148 video-recorded classroom observations across eight middle-grades classes. The eight participating teachers varied in their teaching experience (from two to twenty-one years) and taught in several different school districts (urban, suburban, and rural), so our findings offer a glimpse into a cross section of middle-grades mathematics classrooms. We tabulated the total time spent in various classroom activities and compared that number with the total time observed (92.5 hours). This analysis revealed that going over homework was the most prevalent activity for this diverse group of teachers, surpassing whole-class instruction, group work, and seat work (see the diagram in **fig. 1**).

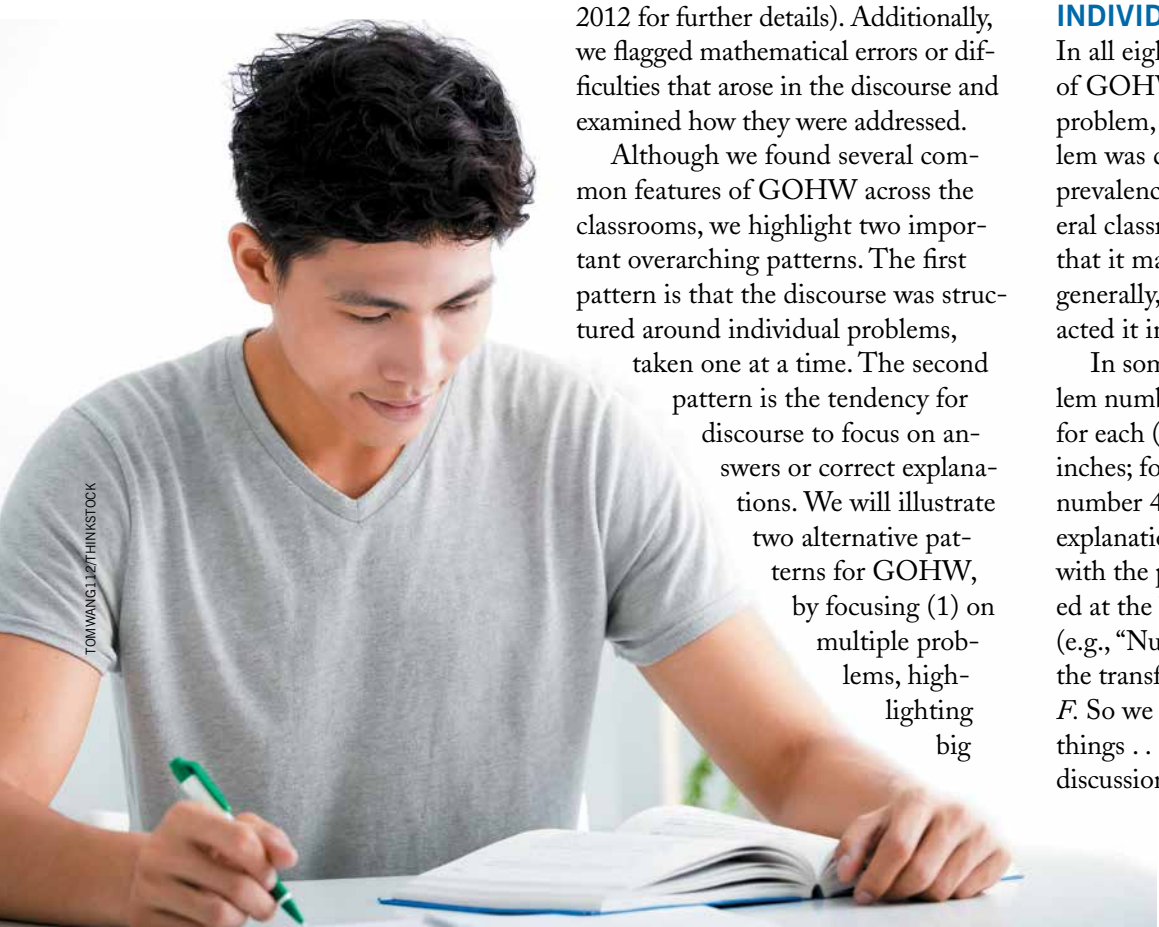
Within the observed lessons, we identified all the instances of GOHW (n = 107) and then considered the ways in which the interactions played out, including how the teachers opened the GOHW activity and how each speaking turn (teacher or student) influenced the nature of the interaction (see Otten, Herbel-Eisenmann, and Cirillo 2012 for further details). Additionally, we flagged mathematical errors or difficulties that arose in the discourse and examined how they were addressed.

Although we found several common features of GOHW across the classrooms, we highlight two important overarching patterns. The first pattern is that the discourse was structured around individual problems, taken one at a time. The second pattern is the tendency for discourse to focus on answers or correct explanations. We will illustrate two alternative patterns for GOHW, by focusing (1) on multiple problems, highlighting big mathematical ideas or questions; and (2) directly on student errors or difficulties. We contend that these alternative patterns are better suited for incorporating the SMP.

Fig. 1 The percentage of class time spent in various classroom activities is clearly delineated.



Source: Adapted from Otten, Herbel-Eisenmann, and Cirillo (2012)



TOM WANG 112/THINKSTOCK

PATTERN 1: TALKING ABOUT INDIVIDUAL PROBLEMS

In all eight classrooms, the discourse of GOHW was structured problem by problem, meaning that only one problem was discussed at one time. The prevalence of this pattern across several classrooms and districts suggests that it may be a feature of GOHW generally, although the teachers enacted it in slightly different ways.

In some cases, teachers stated problem numbers before providing answers for each (e.g., "Number 2 is 75 square inches; for number 3, I had 1400; number 4 is 3.68"). In other cases, explanations were given for problems, with the problem number clearly stated at the beginning of the interaction (e.g., "Number 18: Name and describe the transformation that maps *A* onto *F*. So we have these pointy, hat-shaped things . . ."), with no prompts for discussion across problems.

Focusing on correct solutions may limit the payoff of GOHW because it fails to use incorrect student responses as launching points for discussion, which can be beneficial for student learning.

The problem-by-problem discourse pattern was also evident because of students' responses to teachers' prompts for homework questions. The following example from Mr. Howard's tenth-grade class was typical of the ways the teachers began GOHW:

Mr. Howard: OK, which ones do we need to talk about together from [homework] page 91? Which ones do we want to talk about?
Daniel: 34 and 35
Mr. Howard: 34 and 35. Other requests?
Maria: 48
Will: 31

By phrasing the prompt as "which ones," Mr. Howard subtly shaped the discourse toward a structure based on individual problems. The students' responses to this prompt confirmed the discourse structure as they simply identified individual problems to go over. Even when teachers used more general prompts, such as "Any questions on the homework?" or "What are your questions?" the students still called out individual problem numbers, reaffirming the problem-by-problem discourse pattern.

Although this particular discourse pattern is, in some sense, efficient, it also has major limitations. Students spoke only briefly, although they had considered the problems in advance and could have had much to say. When they did speak, their turns focused only on the problem number rather than on their mathematical thinking. This approach misses opportunities for the SMP that might have been spurred by the homework. It also limits the teachers' opportunities for formative assessment.

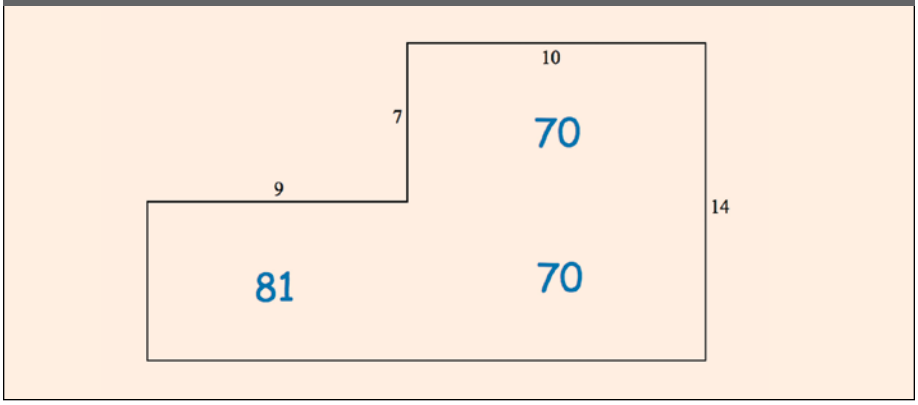
PATTERN 2: TALKING ABOUT CORRECT ANSWERS AND EXPLANATIONS

We found a second pattern within the discourse on individual problems. When students voiced concerns about particular problems, the most common way that teachers responded was to indirectly address the errors or difficulties. By "indirect," we mean that the teachers shared correct solutions or correct explanations that circumvented the error rather than addressing it directly. They explained to the student how to avoid the error or difficulty, which can be viewed as missed opportunities for formative assessment (because the error is not unpacked) and for supporting students to problem solve through an error, as called for by the Standards for Mathematical Practice.

An example of this pattern comes from Mr. Greg's eighth-grade class. The homework assignment involved compound shapes comprising other simple shapes. Mr. Greg began GOHW by having some students write their answers to predetermined problems on the board. He then talked through each of the students' solutions. In one solution, he noticed an error (see **fig. 2**); 81 should have been 63 square units:

Mr. Greg: This [answer] seems off. Because . . . this has a distance of 9, and this distance is 7 going up and down there [the top-middle vertical segment]. Because the total [vertical] length is 14, this length is 7, and so that means this [other length on the far left] has to be 7, because 7 plus 7 equals 14. That makes 63 [for the left rectangle]. Add those together, I think you get 203 for your answer on that. If we used the same method that Brandon showed yesterday, you might have boxed it in and did an overall area of 14 times 19 and come up with 266. This [missing] area is 63, and you subtract away 63 and you still get the same answer. You have a couple of different ways of doing it.

Fig. 2 An erroneous solution written on Mr. Greg's board becomes food for thought.



In this excerpt, Mr. Greg neither asked the student who wrote the incorrect answer to explain his thinking nor other students to critique the reasoning and justify their own conclusions (see SMP 3). He instead explained two correct methods for computing the compound area. His inclusion of multiple solution strategies connects to SMP 1, which involves problem solving, but the students did not necessarily need to persevere or compare the approaches because Mr. Greg provided the explanation.

Focusing on correct solutions may limit the payoff of GOHW because it fails to use incorrect student responses as launching points for discussion, which can be beneficial for student learning (Zahner et al. 2012). Within this discourse pattern, GOHW becomes a mere re-teaching of the lesson that preceded the homework assignment, leading us to the question of how GOHW might be structured differently.

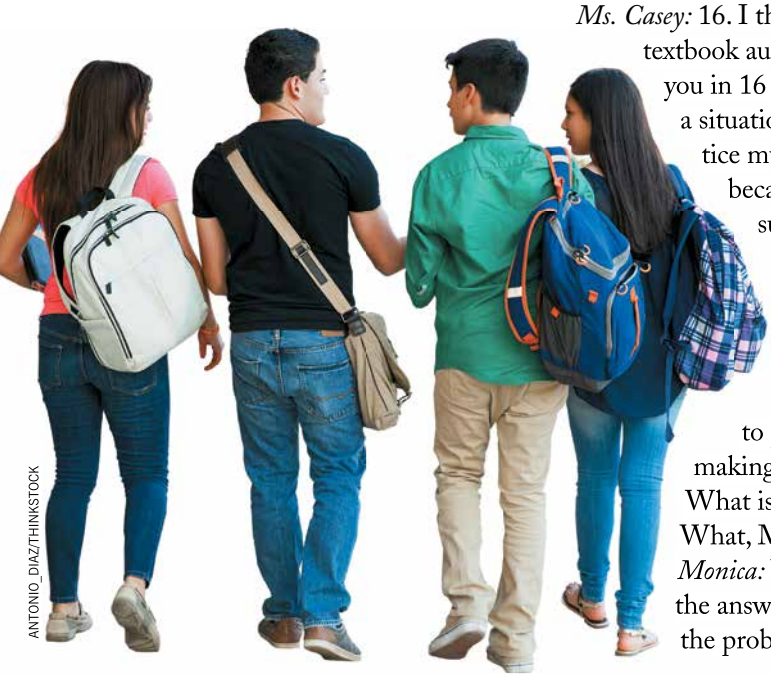
ALTERNATIVE DISCOURSE PATTERNS

In our analysis, we rarely found instances of discourse that deviated from the two patterns described above. Yet, GOHW could be a time for teachers to engage students in the SMP, building on the independent work students have already done on the homework. We provide two alternative patterns to show how this might be accomplished.

One alternative comes from Ms. Casey's seventh-grade classroom and is a rare but illuminating exception to the typical patterns described above. The second pattern is a re-imagining of Mr. Greg's classroom interaction, inspired by prior research on mathematics classroom discourse (Cirillo et al. 2014; Zahner et al. 2012).

ALTERNATIVE PATTERN 1: TALKING ACROSS PROBLEMS

Ms. Casey's homework assignment dealt with the topic of multiplying fractions. The discourse began in the problem-by-problem pattern but moved into a comparing-and-contrasting discussion when problem 16 was discussed, which asked students to find two numbers whose product was between 1/2 and 1. Previous problems on the assignment simply asked students to multiply two given fractions.



ANTONIO_DIAZ/THINKSTOCK

Ms. Casey: 16. I think what [the textbook authors] are doing with you in 16 is they're giving you a situation to let you practice multiplying fractions because that's what we're supposed to be able to do by the end of the chapter. Notice all that we were doing [in previous problems]. You get to practice, but 16 is making you go even further. What is 16 making you do? What, Monica?

Monica: You have to know the answer before you know the problem.

Ms. Casey: Can you describe what you mean by that?

Monica: They want you to give them a problem in which the answer is in between a certain number that they give.

Ms. Casey: So what does that force you to have to do, you guys? Just think about the deciding that has to go on inside your mind to be able to do this. What do you have to do that's different than me just saying, "OK, solve this problem"? [Students raise their hands.] Go ahead.

Chastin: Um, it's different because you are deciding in your head which one would equal that [answer] because you're already trying to equal something, instead of trying to figure out what it equals. You know it has to equal a certain answer, or else the problem's wrong.

Ms. Casey: Would you say it's more complicated?

Chastin: Yes

Ms. Casey: What were you going to say, Kelly?

Kelly: You kind of have to do the problem in your head in order to make sure that it's going to work out.

Ms. Casey: That goes along with Monica. It's like you gotta get the answer to know whether the problem's going to fit. I said it's more complicated a minute ago, but would you agree that they're forcing you to think even more than if they just said, "Solve this problem"? [Students nod.] Deeper thinking, so it's kind of like they're doing good things here. They're making you practice, but they're also making you think more deeply.

Ms. Casey remarked on the purposes of the homework problems and asked students to contrast problem 16 with the previous problems. These moves opened up the discourse, allowing

students to look across problems and talk about how the thinking involved in the problems was different, though related. In essence, number 16 reversed the more straightforward computations in the previous problems and required students to consider constraints involved in selecting appropriate fractions to multiply. Ms. Casey's moves brought this point out during GOHW.

Part of the reason that Ms. Casey was able to make these discourse moves was because the curriculum materials (Lappan et al. 2004) contained a problem that reversed the process of previous tasks. Even without such an example, however, teachers can elicit talk across multiple examples by asking students what was common across sets of problems or how the items in this homework assignment differed from the previous assignment. For example, perhaps there is now an extra step involved or perhaps it is the same idea but it has been applied to a different setting.

Researchers (Jitendra et al. 2009) have found that helping students notice the underlying structures of related mathematics problems and explicitly discussing these structures can positively affect student learning. Such discussions can also lead to rich opportunities to engage in SMP 7 and SMP 8: Looking for and using mathematical structure and expressing regularity in repeated reasoning.

ALTERNATIVE PATTERN 2: TALKING ABOUT ERRORS AND DIFFICULTIES

Directly handling student errors or difficulties arising in GOHW involves making such errors an explicit object of focus. For example, the teacher can—

1. explain why the error is invalid;
2. ask the student to say more about his or her thinking; or
3. ask other students to discuss it.

Table 1 Possible discourse patterns are analyzed for their implications.

Discourse Patterns	Implications
Talking over individual problems, one at a time	• Focus is on the mechanics of one problem rather than big mathematical ideas. • Student discourse is limited to calling out problem numbers or simply describing what students did on a specific problem.
Talking across problems	• Focus is on big mathematical ideas or connections and contrasts between problems. • Student discourse can involve the SMPs, such as problem solving, looking for structure or regularity, or reasoning abstractly.
Talking about correct answers/ explanations	• Focus is on correct thinking. • Ideas and approaches previously taught are repeated. • The importance of correctness in mathematics is emphasized.
Talking about student errors/difficulties	• Focus is on students' actual thinking. • Ideas previously taught may be complemented or clarified. • The importance of careful reasoning and the SMP of persevering, constructing and critiquing arguments, or attending to precision are emphasized.

To imagine how Mr. Greg might have handled the student error directly instead of indirectly, we revisit his classroom example (see **fig. 2**). Because the student wrote his own work on the board, Mr. Greg could have asked him to explain how he determined each of the component areas in the figure. If the student did not recognize the error himself, Mr. Greg could have asked the student probing questions (or asked the rest of the class) about how each dimension of the three smaller rectangles was determined. These moves could help the error surface in the discourse without Mr. Greg simply pointing it out.

Another option was for Mr. Greg to have the students work backward by asking, "If the leftmost rectangle has an area of 81 square units and a width of 9 units, then what must be the length?" This move could eventually help students see the contradiction that the two left vertical segments would sum to 16 units, whereas the right vertical segment was marked as 14 units. Yet another option was for Mr. Greg to ask students if there was an alternative approach they could use to check the work, and then use the resulting interaction to highlight and diagnose the source of the error. In any case, the goal is to make the student error the focus of discussion and a site to promote the SMP and a deeper learning of the material. Moreover, by identifying and working through errors together, going over homework can help students see the process and value of persevering through homework problems.

HOMEWORK AND RICH DISCUSSIONS

Going over homework can be an activity that helps achieve learning goals because students enter the process already having had an opportunity to engage with problems on their own without a specific time limit. Thus, the discourse can build on that work and do much more than just be a review of answers and a time to reiterate suggested solution strategies. Rather, GOHW

can help students see connections and probe into their difficulties as a way of achieving the SMP. **Table 1** summarizes some implications of the discourse of GOHW.

To be clear, we are not suggesting that GOHW must always lead to rich discussions. In some situations, reviewing homework may simply be an item of classroom business that needs to be completed. Nevertheless, it is important to be aware of the common patterns that exist within the GOHW activity, such as talking about only one problem at a time or focusing on answers and correct solutions, because such awareness can lead us to capitalize on alternative ways of spending class time. The alternative patterns we suggested emphasize that mathematics homework—and, by extension, mathematics itself—is not about correct answers, but rather, about reasoning, making connections, and understanding big ideas.

ACKNOWLEDGMENTS

This work was supported by the National Science Foundation (NSF) (grant no. 0347906, Herbel-Eisenmann, principle investigator). Any opinions, findings, and conclusions or recommendations expressed in this article are those of the authors and do not necessarily reflect the views of NSF. We thank the teachers, without whom this research would not have been possible.

CCSSM Practices in Action

- Construct and compare linear, quadratic, and exponential models and solve problems (F.LE.2)
- Interpret expressions for functions in terms of the situation they model (F.LE.5)

REFERENCES

Cirillo, Michelle, Michael D. Steele, Samuel Otten, Beth A. Herbel-Eisenmann, Kathleen McAneny, and Jamila Q. Riser. 2014. "Teacher Discourse Moves: Supporting Productive and Powerful Discourse." In *Using Research to Improve Instruction*, edited by Karen Karp, pp. 141–49. Reston, VA: National Council of Teachers of Mathematics.

Common Core State Standards Initiative (CCSSI). 2010. Common Core State Standards for Mathematics. Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers. http://www.corestandards.org/wp-content/uploads/Math_Standards.pdf

Grouws, Douglas A., James E. Tarr, Ruthmae Sears, and Daniel J. Ross. 2010. "Mathematics Teachers' Use of Instructional Time and Relationships to Textbook Content Organization and Class Period Format." Paper presented at the Hawaii International Conference on Education, Honolulu, Hawaii.

Jitendra, Asha K., Jon R. Star, Kristin

Starosta, Jayne M. Leh, et al. 2009. "Improving Seventh-Grade Students' Learning of Ratio and Proportion: The Role of Schema-Based Instruction." *Contemporary Educational Psychology* 34 (3): 250–64.

Koestler, Courtney, Mathew D. Felton, Kristen N. Bieda, and Samuel Otten. 2013. *Connecting the NCTM Process Standards and the CCSSM Practices.* Reston, VA: NCTM.

Lappan, Glenda, James T. Fey, William M. Fitzgerald, Susan N. Friel, and Elizabeth D. Phillips. 2004. The Connected Mathematics Project. 2nd ed. Menlo Park, CA: Dale Seymour Publications.

National Council of Teachers of Mathematics (NCTM). 2014. *Principles to Actions: Ensuring Mathematics Success for All.* Reston, VA: NCTM.

Otten, Samuel, Beth A. Herbel-Eisenmann, and Michelle Cirillo. 2012. "Going over Homework in Mathematics Classrooms: An Unexamined Activity." *Teachers College Record.* http://www.tcrecord.org/Content.asp?contentid=16851

Smith, Margaret S., Elizabeth K. Hughes, Randi A. Engle, and Mary K. Stein. 2009. "Orchestrating Discussions." *Mathematics Teaching in the Middle School* 14 (May): 548–56.

Webel, Corey. 2010. "Shifting Mathematical Authority from Teacher to Community." *Mathematics Teacher* 104 (November): 315–18.

Wieman, Rob, and Fran Arbaugh. 2014. "Making Homework More Meaningful." *Mathematics Teaching in the Middle School* 20 (October): 160–65.

Zahner, William, Griselda Velazquez, Judit Moschkovich, Phil Vahey, and Teresa Lara-Meloy. 2012. "Mathematics Teaching Practices with Technology That Support Conceptual Understanding for Latino/a Students." *Journal of Mathematical Behavior* 31 (December): 431–46.

Any thoughts on this article? Send an email to **mtms@nctm.org**.—*Ed.*

Samuel Otten, ottensa@missouri.edu, is an assistant professor at the University of Missouri–Columbia. He is interested in students' engagement in such mathematical practices as reasoning and attending to precision. **Michelle Cirillo**, mcirillo@udel.edu, a former high school mathematics teacher, is an associate professor at the University of Delaware. Her interests include classroom discourse and the teaching of such mathematical practices as mathematical modeling and reasoning and proving. **Beth Herbel-Eisenmann**, bhe@msu.edu, is an associate professor at Michigan State University. She is interested in mathematics classroom discourse and collaborating with secondary mathematics teachers to do action research to accomplish their goals for improving classroom discourse.